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Enhancing Inventory Models Using Fuzzy Extended Principles for Optimization

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Abstract

In this work, the authors have considered a physical world concern related to an inventory model under a Neutrosophic environment. We aim to perceive the ideal Total Cost (TC) for the proposed Inventory Management (IM) model without considering the shortage. To achieve this aim, we use trapezoidal Neutrosophic environments. For that purpose, we consider an example to clarify the results of the given model and present computational results. The purpose of the suggested method is to not only solve contemporary numerical problems but also handle new sorts of problems. Finally, we conclude our research by providing graphical, logical, and tabular evaluations with the prevailing methodologies.

Keywords: Neutrosophic inventory model, Trapezoidal fuzzy number, Fuzzy inventory model, Economic order quantity, Inventory management.

1 | Introduction

Operation Research, also known as OR, has a wide scope in mathematics, statistics, decision sciences, and more. The field of OR is very broad and attractive. In operations research, uncertainty is a key factor, and it comes in two main types. The first type, Known Uncertainties, is when we know the chances of different outcomes or events happening. This often requires us to use statistics, models, and risk management, which are common in Operations and Supply Chain Management (OSCM). The second type, Unknowable Unknowns or Black Swans, is when events are completely unexpected and impossible to predict. In real-time applications, uncertainty plays an important role, such that it could help us learn more and improve how we understand and manage operations. In our study, we have identified that uncertainty is the predominant challenge across all these applications, as illustrated in *Table 1* below.

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Table 1. Investigating the wide-ranging uses of OR in different domains.

S/N	Ref.	Implementation	Uncertain Environments	Significance
1	[1]	Data modeling	Neutrosophic	A DEA analysis from a Neutrosophic viewpoint.
2	[2]	Linear programming	Neutrosophic	Implementing a nonlinear methodology in Neutrosophic linear programming.
3	[3]	Shortest path problems (SPP)	Weighted Fuzzy	Addressing problems related to the shortest path utilizing fuzzy weighted arc length.
4	[4]	AFS	Fuzzy	Discussing a concept of Anti-Fuzzy Subgroups (AFS) and their various attributes.
5	[5]	SPP	Fuzzy	Observations on the approach to select the optimal path for a reliable SPP under fuzzy conditions.
6	[6]	Decision-making	Fuzzy	Improved a new method for making data smoother in situations where we're creating models or making decisions and there's some uncertainty involved.
7	[7]	Complex programming	Fuzzy	Working on identifying solutions for complicated coding problems that involve uncertain elements in the limitations.
8	[8]	Goal programming	Fuzzy	Using the min-max goal programming method to solve problems that involve multiple objectives, where the objectives are not fixed but are determined anew (De Novo).
9	[9]	Fermatean SPP	Neutrosophic	Solving the SPP using a range of fuzzy arc lengths, based on Fermatean Neutrosophic logic: this involves creating a formula and modifying Dijkstra's algorithm.

As mentioned in *Table 1* above, there is uncertainty involved in some fields of OR. However, my main focus is to explain the uncertainty that arises in Inventory Management (IM). That's why we are now going to discuss IM.

APICS Dictionary [10], stocks and items used to support activities, customer service, and production as related to the IM. The stocks or items include raw materials, work-in-process items, maintenance, repair and operating supplies, finished goods, and spare parts. Inventory control is essential for many reasons in a manufacturing organization. One of the reasons is money, which is tied up in inventory. Therefore, inventory control is very important for research purposes and real-world applications. The EOQ model is mostly used in inventory control. For successive operations, the EOQ model classifies demand and supply. The simple EOQ was used for the first quantitative treatment of inventory.

Harris developed this model in [11]. Wilson [12] used the EOQ model in academics and industries, which increased interest in this model. By using the basic EOQ model, the study aims to identify and minimize the holding inventory and placing orders for the fixed setup, which is analyzed based on its order size. The assumptions made by this model include that the demand is known, fixed, and independent; quantity discounts are not allowed; inventory replenishment is instantaneous, and no safety stock is available. In recent years, several notable contributions have been made to the field of IM. Some IM models have been developed by Rabta [13], Biuki et al. [14], Das et al. [15], and Utama [16]. Rabta's model [13] is based on the economic

order quantity and is tailored for a product that incorporates a circular economy indicator. Biuki et al.'s model [14] is comprehensive and integrates routing, location, and inventory considerations for the environmental design of a supply chain network specializing in perishable products.

Das et al.'s model [15] incorporates a partial trade credit policy and reliability considerations. Namakshenas and Mahdavi Mazdeh [17] proposed and developed a plan to manage the production and storage of quickly expiring medicines in a medical imaging center. Utama's model [16] is a comprehensive analysis of a procurement-production IM model that is incorporated into the supply chain environment. Abouhawwash et al. [18] advanced a system using artificial intelligence to predict CO2 levels, which has helped to manage environmental sustainability.

San-Jose et al. [19], in order to manage inventory for items that perish over time, considered power usage and allowed for backorders while accounting for a carbon tax. Rodriguez-Mazo and Ruiz-Benitez [20] progressed on the plan to manage inventory across multiple warehouses, where demand depends on both price and stock levels, and items are released in large batches. Sebatjane and Adetunji [21] proposed a supply chain model with four levels for items that grow, considering imperfect quality and inspection mistakes. Classical IM faces many challenges. In fact, there are several parameters, such as demand, holding cost, and set-up cost, that are used in the classical model. However, uncertainty has a significant impact on all of these parameters. As a result, the classical model fails to predict the best solution.

As a result, Zadeh [22] introduced the fuzzy theory in 1965, recognizing its potential for enhancing decision-making processes. Lately, Dubois and Parade [23] introduced a new inventory model called the fuzzy inventory model. Several researchers have studied the significant shift in the approach to IM and supply chain.

Mohamed et al. [24] used fuzzy logic and maths to improve healthcare by choosing the best blockchain provider. Moreover, Mohamed et al. [25] again used fuzzy logic and other related techniques to choose the best smart logistics company using IoT, blockchain, and drones. Finally, Jain et al. [26] used trapezoidal fuzzy numbers to solve inventory problems in decision science.

Traditional inventory models assume precise and deterministic parameters, but the fuzzy inventory model allows for the representation of vague and ambiguous information. This enables decision-makers to make more robust and flexible IM decisions. Later on, to handle uncertainty better, fuzzy extensions [27] (such as intuitionistic fuzzy [28], [29], Pythagorean fuzzy [30], [31], and Neutrosophic set [32–35], were developed. However, Smarandache [35] proposed the concept of an NS. He highlighted some of the specific characteristics that make it superior to the classical model and fuzzy model.

The objective and novelty

After extensively reviewing academic texts and scholarly articles, it is evident that possessing a substantial understanding of the field of management is of utmost significance. Furthermore, it is crucial to comprehend that uncertainty plays a significant role in physical world issues. In this context, the objective of this research is to investigate NIM without shortage. The study intends to investigate NIM without shortage. In order to accomplish this, the following goals have been defined:

I. Introducing a novel approach to handle and control uncertainty efficiently in the context of IM.

II. Evaluating the proposed approach in relation to current methodologies to determine its viability.

III. Examine the real-world implementation of the developed approach in the IM system.

IV. Performing an extensive literature review on IM systems to improve comprehension of the complete management paradigm.

The present work aims to expand the existing understanding of the discipline by overcoming the highlighted issues and providing novel ideas to improve the effectiveness of IM systems in uncertain situations. In light of this, the paper has been classified into six parts. The first part is the introduction. The second part offers

explicit definitions and introductory content. The third part examines notations, assumptions, and the current state of classical IM.

The fourth part introduces the IM developed for Neutrosophic environments. The fifth part encompasses numerical illustrations and outlines the benefits and constraints of the proposed model in comparison to other current models. Ultimately, the final part summarizes the work.

Abbreviations

I. MF presents for membership function.

II. TrFN presents a triangular fuzzy number.

III. TpFN presents for trapezoidal fuzzy number.

IV. SVTpN presents for single-valued trapezoidal Neutrosophic.

V. TpNNs present for trapezoidal Neutrosophic numbers.

2 | Definitions and Preliminaries

Definition 1. ([26]). Let $\lambda_{\tilde{j}}, \pi_{\tilde{j}}, \rho_{\tilde{j}} \in [0,1]$, then, an SVTpN number $\tilde{j} = \left\langle \left[\tilde{j}^a, \tilde{j}^s, \tilde{j}^h, \tilde{j}^o\right], (\lambda_{\tilde{j}}, \pi_{\tilde{j}}, \rho_{\tilde{j}}) \right\rangle$ is a Neutrosophic set on R, whose MF of truth $\zeta_{\tilde{j}}(x)$, MF of indeterminacy $\psi_{\tilde{j}}(x)$, and MF of falsity $\xi_{\tilde{j}}(x)$ are given as follows:

$$\zeta_{\tilde{j}}(x) = \begin{cases} \dfrac{\lambda_{\tilde{j}}(x - \tilde{j}^a)}{(\tilde{j}^s - \tilde{j}^a)}, & \tilde{j}^a \le x \le \tilde{j}^s \\ \lambda_{\tilde{j}}, & \tilde{j}^s \le x \le \tilde{j}^h \\ \dfrac{\lambda_{\tilde{j}}(\tilde{j}^o - x)}{(\tilde{j}^o - \tilde{j}^h)}, & \tilde{j}^h \le x \le \tilde{j}^o \\ 0, & \text{otherwise} \end{cases}.$$

$$\psi_{\tilde{j}}(x) = \begin{cases} \dfrac{(\tilde{j}^s - x + \pi_{\tilde{j}}(x - \tilde{j}^a))}{(\tilde{j}^s - \tilde{j}^a)}, & \tilde{j}^a \le x \le \tilde{j}^s \\ \pi_{\tilde{j}}, & \tilde{j}^s \le x \le \tilde{j}^h \\ \dfrac{(x - \tilde{j}^h + \pi_{\tilde{j}}(\tilde{j}^o - x))}{(\tilde{j}^o - \tilde{j}^h)}, & \tilde{j}^h \le x \le \tilde{j}^o \\ 1, & \text{otherwise} \end{cases}.$$

$$\xi_{\tilde{j}}(x) = \begin{cases} \dfrac{(\tilde{j}^s - x + \rho_{\tilde{j}}(x - \tilde{j}^a))}{(\tilde{j}^s - \tilde{j}^a)}, & \tilde{j}^a \le x \le \tilde{j}^s \\ \rho_{\tilde{j}}, & \tilde{j}^s \le x \le \tilde{j}^h \\ \dfrac{(x - \tilde{j}^h + \rho_{\tilde{j}}(\tilde{j}^o - x))}{(\tilde{j}^o - \tilde{j}^h)}, & \tilde{j}^h \le x \le \tilde{j}^o \\ 1, & \text{otherwise} \end{cases}.$$

Definition 2. ([36]). Let $\underset{\sim}{J} = \left\langle [\tilde{j}_1^a, \tilde{j}_2^n, \tilde{j}_3^k, \tilde{j}_4^i]; (\rho_{\underset{\sim}{J}}, \lambda_{\underset{\sim}{J}}, \pi_{\underset{\sim}{J}}) \right\rangle$ and $\underset{\sim}{N} = \left\langle [\tilde{n}_1^a, \tilde{n}_2^n, \tilde{n}_3^k, \tilde{n}_4^i]; (\rho_{\underset{\sim}{N}}, \lambda_{\underset{\sim}{N}}, \pi_{\underset{\sim}{N}}) \right\rangle$ are two TpNNs, then

I. $\underset{\sim}{J} \otimes \underset{\sim}{N} = \left\langle [\tilde{j}_1^a\tilde{n}_1^a, \tilde{j}_2^n\tilde{n}_2^n, \tilde{j}_3^k\tilde{n}_3^k, \tilde{j}_4^i\tilde{n}_4^i]; (\rho_{\underset{\sim}{J}} \wedge \rho_{\underset{\sim}{B}}, \lambda_{\underset{\sim}{J}} \vee \lambda_{\underset{\sim}{B}}, \pi_{\underset{\sim}{J}} \vee \pi_{\underset{\sim}{B}}) \right\rangle.$

II. $\alpha \otimes \underset{\sim}{J} = \left\langle [\alpha\tilde{j}_1^a, \alpha\tilde{j}_2^n, \alpha\tilde{j}_3^k, \alpha\tilde{j}_4^i]; (\rho_{\underset{\sim}{J}}, \lambda_{\underset{\sim}{J}}, \pi_{\underset{\sim}{J}}) \right\rangle, \alpha \ge 0.$

III. $\alpha \otimes \underset{\sim}{J} = \left\langle [\alpha \tilde{j}_4^i, \alpha \tilde{j}_3^k, \alpha \tilde{j}_2^n, \alpha \tilde{j}_1^a]; (\rho_{\underset{\sim}{J}}, \lambda_{\underset{\sim}{J}}, \pi_{\underset{\sim}{J}}) \right\rangle, \alpha < 0.$

IV. $\underset{\sim}{J} \oplus \underset{\sim}{N} = \left\langle [\tilde{j}_1^a + \tilde{n}_1^a, \tilde{j}_2^n + \tilde{n}_2^n, \tilde{j}_3^k + \tilde{n}_3^k, \tilde{j}_4^i + \tilde{n}_4^i]; (\rho_{\underset{\sim}{J}} \wedge \rho_{\underset{\sim}{N}}, \lambda_{\underset{\sim}{J}} \vee \lambda_{\underset{\sim}{N}}, \pi_{\underset{\sim}{J}} \vee \pi_{\underset{\sim}{N}}) \right\rangle.$

Definition 3. ([37]). Let $\underset{\sim}{J} = \left\langle [\tilde{j}_1^a, \tilde{j}_2^n, \tilde{j}_3^k, \tilde{j}_4^i]; (\rho_{\underset{\sim}{J}}, \lambda_{\underset{\sim}{J}}, \pi_{\underset{\sim}{J}}) \right\rangle$ and $\underset{\sim}{N} = \left\langle [\tilde{n}_1^a, \tilde{n}_2^n, \tilde{n}_3^k, \tilde{n}_4^i]; (\rho_{\underset{\sim}{N}}, \lambda_{\underset{\sim}{N}}, \pi_{\underset{\sim}{N}}) \right\rangle$ be two SVTpN numbers. Then,

I. If $Sc(\underset{\sim}{J}) < Sc(\underset{\sim}{N})$, then $\underset{\sim}{J}$ is smaller than $\underset{\sim}{N}$, denoted by $\underset{\sim}{J} < \underset{\sim}{N}$.

II. If $Sc(\underset{\sim}{J}) = Sc(\underset{\sim}{N})$;

- *If* $Ac(\underset{\sim}{J}) < Ac(\underset{\sim}{N})$, *then* $\underset{\sim}{J}$ *is smaller than* $\underset{\sim}{N}$, *denoted by* $\underset{\sim}{J} < \underset{\sim}{N}$.
- *If* $Ac(\underset{\sim}{J}) = Ac(\underset{\sim}{N})$, *then* $\underset{\sim}{J}$ *and* $\underset{\sim}{N}$ *are the same, denoted by* $\underset{\sim}{J} = \underset{\sim}{N}$,

where, $\underset{\sim}{J} = \left\langle [\tilde{j}_1^a, \tilde{j}_2^n, \tilde{j}_3^k, \tilde{j}_4^i]; (\rho_{\underset{\sim}{J}}, \lambda_{\underset{\sim}{J}}, \pi_{\underset{\sim}{J}}) \right\rangle$ be TpFN then the score function of $\underset{\sim}{J}$ is defined as

$$Sc(\underset{\sim}{J}) = \frac{1}{12}(\tilde{j}_1^a + \tilde{j}_2^n + \tilde{j}_3^k + \tilde{j}_4^i)(2 + \rho_{\underset{\sim}{J}} - \lambda_{\underset{\sim}{J}} - \pi_{\underset{\sim}{J}}).$$

3 | Symbolizations, Assumptions, and Existing Classical IM

Before we move forward with our suggested model, we have discussed symbolizations, assumptions, and existing classical IM in the following subsections.

Symbols

i	Order quantity per cycle.
k	Length of the plan.
n	Set up cost per order.
d	Time planning period of the demand $[0,k]$.
$\widehat{n}_{neu}$	Neutrosophic ordering cost.
tc	Cost of the period $[0,k]$.
$\widehat{a}_{neu}$	Neutrosophic carrying cost.
$\hat{i}_{neu}$	Neutrosophic EOQ.
a	Holding cost per unit quantity per unit time.
t	Length of the cycle.

Assumptions

In this technique, there are some assumptions are considered:

I. Shortages are not allowed.

II. The time of the plan is constant.

III. Only carrying and ordering costs are Neutrosophic in nature.

IV. Total demand is considered constant.

Formulation of IM in a classical environment

According to the IM model without shortage in crisp, the EOQ is represented in the below equation:

$$i = \sqrt{\frac{2.n.d}{ak}}. \quad (1)$$

The Total Cost (TC) for the period $[0,k]$,

$$tc = \frac{aki}{2} + \frac{nd}{i}. \tag{2}$$

Now differentiate *Eq (2)* and obtain the optimum $i^* \,\&\, tc^*$.

$$i^* = \sqrt{\frac{2.n.d}{ak}}. \tag{3}$$

$$tc^* = \sqrt{2nadk}. \tag{4}$$

4 | Proposed Inventory Model in Neutrosophic Sense

By considering the Neutrosophic environment, the features of carrying cost and set-up cost change, and the representation likely taken as TpNNs.

Time and the total amount of demand have been considered to be constant, further *Eq. (2)* represents the Neutrosophic cost parameter:

$$t\hat{c}_{neu} = \frac{\hat{a}_{neu} ki}{2} + \frac{\hat{n}_{neu} d}{i}. \tag{5}$$

Suppose $\hat{a}_{neu} = \begin{pmatrix} \langle \hat{a}^1_{neu}, \hat{a}^2_{neu}, \hat{a}^3_{neu}, \hat{a}^4_{neu} \rangle; \\ (\mu_{\hat{a}_{neu}}, \sigma_{\hat{a}_{neu}}, \omega_{\hat{a}_{neu}}) \end{pmatrix} \& \, \hat{n}_{neu} = \begin{pmatrix} \langle \hat{n}^1_{neu}, \hat{n}^2_{neu}, \hat{n}^3_{neu}, \hat{n}^4_{neu} \rangle; \\ (\mu_{\hat{n}_{neu}}, \sigma_{\hat{n}_{neu}}, \omega_{\hat{n}_{neu}}) \end{pmatrix}$ are Neutrosophic trapezoidal carrying and ordering numbers, then from *Eq. (5)*, we have

$$t\hat{c}_{neu} = t\hat{c}_{neu}(\hat{a}_{neu}, \hat{n}_{neu}) = \left[\hat{a}_{neu} . \frac{ki}{2}\right] + \left[\hat{n}_{neu} . \frac{d}{i}\right].$$

$$t\hat{c}_{neu} = \left[\begin{pmatrix} \langle \hat{a}^1_{neu}, \hat{a}^2_{neu}, \hat{a}^3_{neu}, \hat{a}^4_{neu} \rangle; \\ (\mu_{\hat{a}_{neu}}, \sigma_{\hat{a}_{neu}}, \omega_{\hat{a}_{neu}}) \end{pmatrix} . \frac{ki}{2}\right] + \left[\begin{pmatrix} \langle \hat{n}^1_{neu}, \hat{n}^2_{neu}, \hat{n}^3_{neu}, \hat{n}^4_{neu} \rangle; \\ (\mu_{\hat{n}_{neu}}, \sigma_{\hat{n}_{neu}}, \omega_{\hat{n}_{neu}}) \end{pmatrix} . \frac{d}{i}\right].$$

$$t\hat{c}_{neu} = \left[\begin{matrix} \left(\hat{a}^1_{neu} . \frac{ki}{2}, \hat{a}^2_{neu} . \frac{ki}{2}, \hat{a}^3_{neu} . \frac{ki}{2}, \hat{a}^4_{neu} . \frac{ki}{2}\right); \\ (\mu_{\hat{a}_{neu}}, \sigma_{\hat{a}_{neu}}, \omega_{\hat{a}_{neu}}) \end{matrix}\right] + \left[\begin{matrix} \left(\hat{n}^1_{neu} . \frac{d}{i}, \hat{n}^2_{neu} . \frac{d}{i}, \hat{n}^3_{neu} . \frac{d}{i}, \hat{n}^4_{neu} . \frac{d}{i}\right); \\ (\mu_{\hat{n}_{neu}}, \sigma_{\hat{n}_{neu}}, \omega_{\hat{n}_{neu}}) \end{matrix}\right].$$

By using def 2 [34] and def [35] we have

$$[t\hat{c}_{neu}] = \frac{1}{4}\left[\begin{pmatrix} \hat{a}^1_{neu} . \frac{ki}{2} + \hat{n}^1_{neu} . \frac{d}{i} + \hat{a}^2_{neu} . \frac{ki}{2} + \hat{n}^2_{neu} . \frac{d}{i} \\ + \hat{a}^3_{neu} . \frac{ki}{2} + \hat{n}^3_{neu} . \frac{d}{i} + \hat{a}^4_{neu} . \frac{ki}{2} + \hat{n}^4_{neu} . \frac{d}{i} \end{pmatrix}\right]\left[\frac{\left(2 + (\mu_{\hat{a}_{neu}} \wedge \mu_{\hat{n}_{neu}}) - (\sigma_{\hat{a}_{neu}} \vee \sigma_{\hat{n}_{neu}}) - (\omega_{\hat{a}_{neu}} \vee \omega_{\hat{n}_{neu}})\right)}{3}\right].$$

$$[t\hat{c}_{neu}] = \frac{1}{4}\left[\begin{pmatrix} \left(\hat{a}^1_{neu} + \hat{a}^2_{neu} + \hat{a}^3_{neu} + \hat{a}^4_{neu}\right) . \frac{ki}{2} \\ + \left(\hat{n}^1_{neu} + + \hat{n}^2_{neu} + \hat{n}^3_{neu} + \hat{n}^4_{neu}\right) . \frac{d}{i} \end{pmatrix}\right]\left[\frac{\left(2 + (\mu_{\hat{a}_{neu}} \wedge \mu_{\hat{n}_{neu}}) - (\sigma_{\hat{a}_{neu}} \vee \sigma_{\hat{n}_{neu}}) - (\omega_{\hat{a}_{neu}} \vee \omega_{\hat{n}_{neu}})\right)}{3}\right] = \varphi(i). \tag{6}$$

$\varphi(i)$ is minimum when $\frac{d\varphi(i)}{di} = 0, \&\text{where} \frac{d^2\varphi(i)}{di^2} > 0.$

Now, $\frac{d\varphi(i)}{di} = 0$ gives the economic order quantity as

$$\frac{1}{4}\left[\left(\begin{array}{l}\left(\hat{a}^1_{neu} + \hat{a}^2_{neu} + \hat{a}^3_{neu} + \hat{a}^4_{neu}\right).\frac{k}{2}\\ +\left(\hat{n}^1_{neu} + +\hat{n}^2_{neu} + \hat{n}^3_{neu} + \hat{n}^4_{neu}\right).\left(-\frac{d}{i^2}\right)\end{array}\right)\right]\left[\frac{\left(2 + (\mu_{\hat{a}_{neu}} \wedge \mu_{\hat{n}_{neu}}) - (\sigma_{\hat{a}_{neu}} \vee \sigma_{\hat{n}_{neu}}) - (\omega_{\hat{a}_{neu}} \vee \omega_{\hat{n}_{neu}})\right)}{3}\right] = 0.$$

$$i^2 = \frac{2d}{k}\frac{\left(\hat{n}^1_{neu} + \hat{n}^2_{neu} + \hat{n}^3_{neu} + \hat{n}^4_{neu}\right)}{\left(\hat{a}^1_{neu} + \hat{a}^2_{neu} + \hat{a}^3_{neu} + \hat{a}^4_{neu}\right)}.$$

$$i = \sqrt{\frac{2d}{k}\frac{\left(\hat{n}^1_{neu} + \hat{n}^2_{neu} + \hat{n}^3_{neu} + \hat{n}^4_{neu}\right)}{\left(\hat{a}^1_{neu} + \hat{a}^2_{neu} + \hat{a}^3_{neu} + \hat{a}^4_{neu}\right)}}.$$

Therefore, the Neutrosophic optimal quantity is

$$\hat{i}_{neu} = \sqrt{\frac{2d}{k}\frac{\left(\hat{n}^1_{neu} + \hat{n}^2_{neu} + \hat{n}^3_{neu} + \hat{n}^4_{neu}\right)}{\left(\hat{a}^1_{neu} + \hat{a}^2_{neu} + \hat{a}^3_{neu} + \hat{a}^4_{neu}\right)}}. \quad \textbf{(7)}$$

Also, at $i = \hat{i}_{neu}$, we have $\frac{d^2\varphi(i)}{di^2} > 0$, this shows that $\varphi(i)$ is minimum at $i = \hat{i}_{neu}$. And from *Eq. (6)*, we find:

$$[\varphi(i)]^* = \frac{1}{4}\left[\left(\begin{array}{l}\left(\hat{a}^1_{neu} + \hat{a}^2_{neu} + \hat{a}^3_{neu} + \hat{a}^4_{neu}\right).\frac{ki}{2}\\ +\left(\hat{n}^1_{neu} + \hat{n}^2_{neu} + \hat{n}^3_{neu} + \hat{n}^4_{neu}\right).\frac{d}{i}\end{array}\right)\right]\left[\frac{\left(2 + (\mu_{\hat{a}_{neu}} \wedge \mu_{\hat{n}_{neu}}) - (\sigma_{\hat{a}_{neu}} \vee \sigma_{\hat{n}_{neu}}) - (\omega_{\hat{a}_{neu}} \vee \omega_{\hat{n}_{neu}})\right)}{3}\right]. \quad \textbf{(8)}$$

5 | Numerical Example

In this section, we explain how we validated our proposed model. We also compared our model with existing ones to demonstrate its superiority. To demonstrate this, we have carried out two case studies employing existing literature databases, which encompassed the work of Kumar and Dutta [38]. In illustration one, we delineate our methodology by comparing it against the presently existing techniques. Furthermore, illustration two demonstrates that the suggested approach not only tackles contemporary issues but all together resolves a novel form of environment. Moreover, in subsection 5.1, we discussed the advantages, and in subsection 5.2, we also talked about the limitations.

Example 1. ([38], [39]). A farmer uses 500 units of rice in his storage house, which costs 1.2rs/unit. The inventory has a carrying cost of 10% and an order cost of 20/day. Find the EOQ and the total inventory for 6 days in both crisp and Neutrosophic sense. We have observed that there is a lot of uncertainty in holding cost and set-up cost, which is why we have transformed them into a Neutrosophic environment. Where holding cost $\hat{a}_{neu} = \langle [8,11,13,16];(1,0,0)\rangle$, and set up cost $\hat{n}_{neu} = \langle [15,19,21,24];(1,0,0)\rangle$.

Solution: in *Example 1*, we're showing that our new method not only solves a novel problem but also handles the issue addressed by the current technique. This can be observed in *Table 2* provided below. More precisely, we note that the TC of our suggested approach is equivalent to the current techniques.

Table 2. Analysis while considering holding cost ($\hat{a}_{neu}$) and set up costs ($\hat{n}_{neu}$) under a Neutrosophic environment.

	Total Cost		
Demand	**Classical Environment**	**Kumar and Dutta [38]**	**Proposed Model**
500	1200	1192.476	1192.476
600	1314.534	1306.292	1306.292
700	1419.859	1410.957	1410.957
800	1517.893	1508.377	1508.377
900	1609.969	1599.875	1599.875
1000	1697.056	1686.416	1686.416

Besides comparing *Table 2*, we have also compared the graph with Kumar and Dutta [38] in *Fig. 1*.

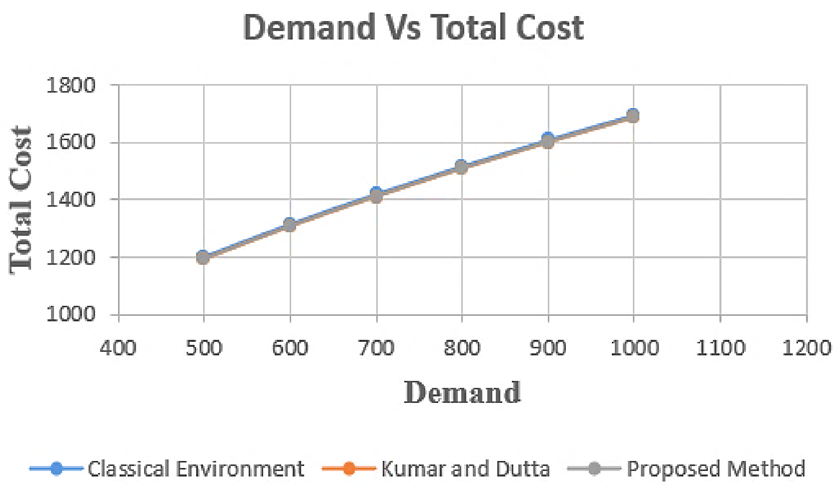

Fig. 1. A graphical comparison study with Kumar and Dutta [38].

In *Example 1*, several authors have suggested various approaches to address the numerical problem posed by Kumar and Dutta [38] When comparing the tabular and pictorial methods, we found that our proposed solution closely aligns with Kumar and Dutta [38] optimal solution. Additionally, our approach not only addresses current issues yet handles a novel sort of environment, as illustrated in *Example 2*.

Example 2. A farmer uses 500-900 units of rice in his storage house, which costs 1.2rs/unit. The inventory has a carrying cost of 10 percent and an order cost of 20/day. Find the EOQ and the total inventory for 6 days in both classical and Neutrosophic sense. We have observed that there is a lot of uncertainty in holding cost and set-up cost, which is why we have transformed them into a Neutrosophic environment, where we consider the following cases 1-5.

Let us consider the following data shown in *Table 3* for solving our proposed model.

Table 3. Data for solving our proposed model.

Different Cases	**setup cost =** $\hat{n}_{neu} = \left(\left\langle \hat{n}^1_{neu}, \hat{n}^2_{neu}, \hat{n}^3_{neu}, \hat{n}^4_{neu} \right\rangle; \left(\mu_{\hat{n}_{neu}}, \sigma_{\hat{n}_{neu}}, \omega_{\hat{n}_{neu}}\right)\right)$**, Carrying cost =** $\hat{a}_{neu} = \left(\left\langle \hat{a}^1_{neu}, \hat{a}^2_{neu}, \hat{a}^3_{neu}, \hat{a}^4_{neu} \right\rangle; \left(\mu_{\hat{a}_{neu}}, \sigma_{\hat{a}_{neu}}, \omega_{\hat{a}_{neu}}\right)\right)$ **, and Demand (d).**
Case (I)	$\left\langle (19,27,31,36);(0.98,0.65,0.58) \right\rangle$, $\left\langle (12,19,23,28);(0.94,0.78,0.19) \right\rangle$, 500.
Case (II)	$\left\langle (25,39,46,54);(0.89,0.45,0.19) \right\rangle$, $\left\langle (18,31,38,46);(0.75,0.56,0.33) \right\rangle$, 600.
Case (III)	$\left\langle (27,43,51,60);(0.78,0.65,0.45) \right\rangle$, $\left\langle (20,35,43,52);(0.78,0.77,0.56) \right\rangle$, 700.
Case (IV)	$\left\langle (29,47,56,66);(0.73,0.35,0.11) \right\rangle$, $\left\langle (22,39,48,58);(0.97,0.59,0.37) \right\rangle$, 800.
Case (V)	$\left\langle (31,51,61,72);(0.96,0.65,0.23) \right\rangle$, $\left\langle (24,43,53,64);(0.99,0.49,0.13) \right\rangle$, 900.

Solution: we are here to provide a solution for an example that our proposed method can solve. However, some existing methods, such as De and Rawat [37] and Kumar and Dutta [36], cannot solve it. This has been demonstrated in *Table 4* below.

Table 4. Finding the total ideal cost for different situations.

Different Cases	Proposed Method
Case (I)	981.7425
Case (II)	1942.431
Case (III)	1824.783
Case (IV)	2627.958
Case (V)	3582.796

Besides comparing *Table 4*, we have also compared pictures with some existing methods, such as De and Rawat [39] and Kumar and Dutta [38] in *Fig. 2*.

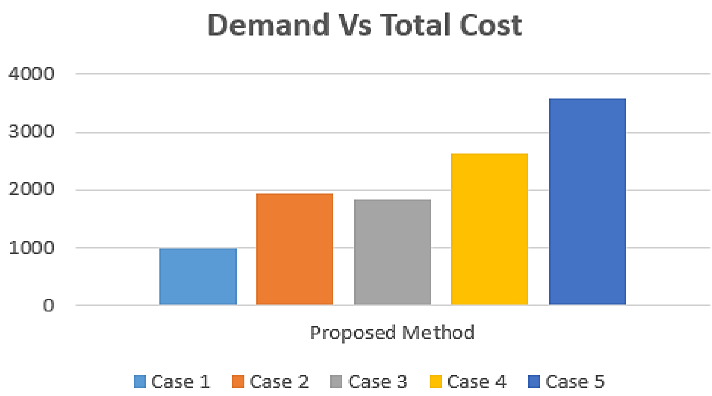

Fig. 2. A visual comparative analysis using existing examples.

From *Table 2*, we observed that the EOQ is closer to the crisp EOQ, and the TC is less than the crisp TC. We can also see from *Table 2* that our proposed method solves a new problem, and at the same time, it solves the problem of the existing method, i.e., Kumar and Dutta [38]. Later on, we took *Example 2*, whose data are shown in the table. In *Table 4*, we have explained the solution to the problem of *Example 2*, and we observed that our proposed method is the only one that can solve this new problem, while the methods of De and Rawat [39] and Kumar and Dutta [38] cannot. We can also observe that the EOQ is more sensitive, as the TC is directly proportional to the demand.

5.1 | Advantages

I. The suggested approach can effortlessly determine the overall expense and the best amount to order.

II. It simplifies the time required for computation.

III. It is more dependable and better than all other current methods.

IV. It addresses the shortcomings of several current methods.

5.2 | Limitations

I. The model does not take into account the possibility of a shortage in the inventory.

II. The model is developed and tested under trapezoidal Neutrosophic environments.

6 | Conclusion

The presented study evolves a novel IM under a Neutrosophic environment, which can handle the uncertainty and imprecision of real-life situations. We have used trapezoidal Neutrosophic numbers to represent the demand, holding cost, and ordering cost of the inventory system. The authors have formulated the most

efficient formula for calculating the TC and successfully applied it to a numerical illustration to highlight the practicality and efficiency of the suggested approach. In addition, the authors have evaluated to assess the influence of various parameters on the ideal solution. The comparison results with the existing methods have found that our proposed model is more flexible, realistic, and efficient. Our model can be extended to other inventory problems with different assumptions and constraints.

Author Contribution

For research articles, the authors provide a short paragraph that identifies each contribution. Ankit Dubey: methodology, software, conceptualization, writing. Ranjan Kumar: editing, supervising, writing. All authors have read and agreed to the published version of the manuscript.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings.

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